Estimation of Parsimonious Covariance Models for Gaussian Matrix Valued Random Variables for Multi-Dimensional Spectroscopic Data

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Introduction

Satellite remote sensing makes it possible to observe landscapes on large spatial scales. The Sentinel-1 and Sentinel-2 satellites currently provide full coverage of the national territory of France every 5 days. Due to the orbit of the satellites, coupled with the presence of clouds, the sampling of the pixels are temporally irregular. **The project aims to develop, study and implement supervised and unsupervised classification methods when the data are of different natures (heterogeneous) and have missing and / or aberrant data.** The methods implemented are developed to process satellite and aerial data for ecology and cartography.

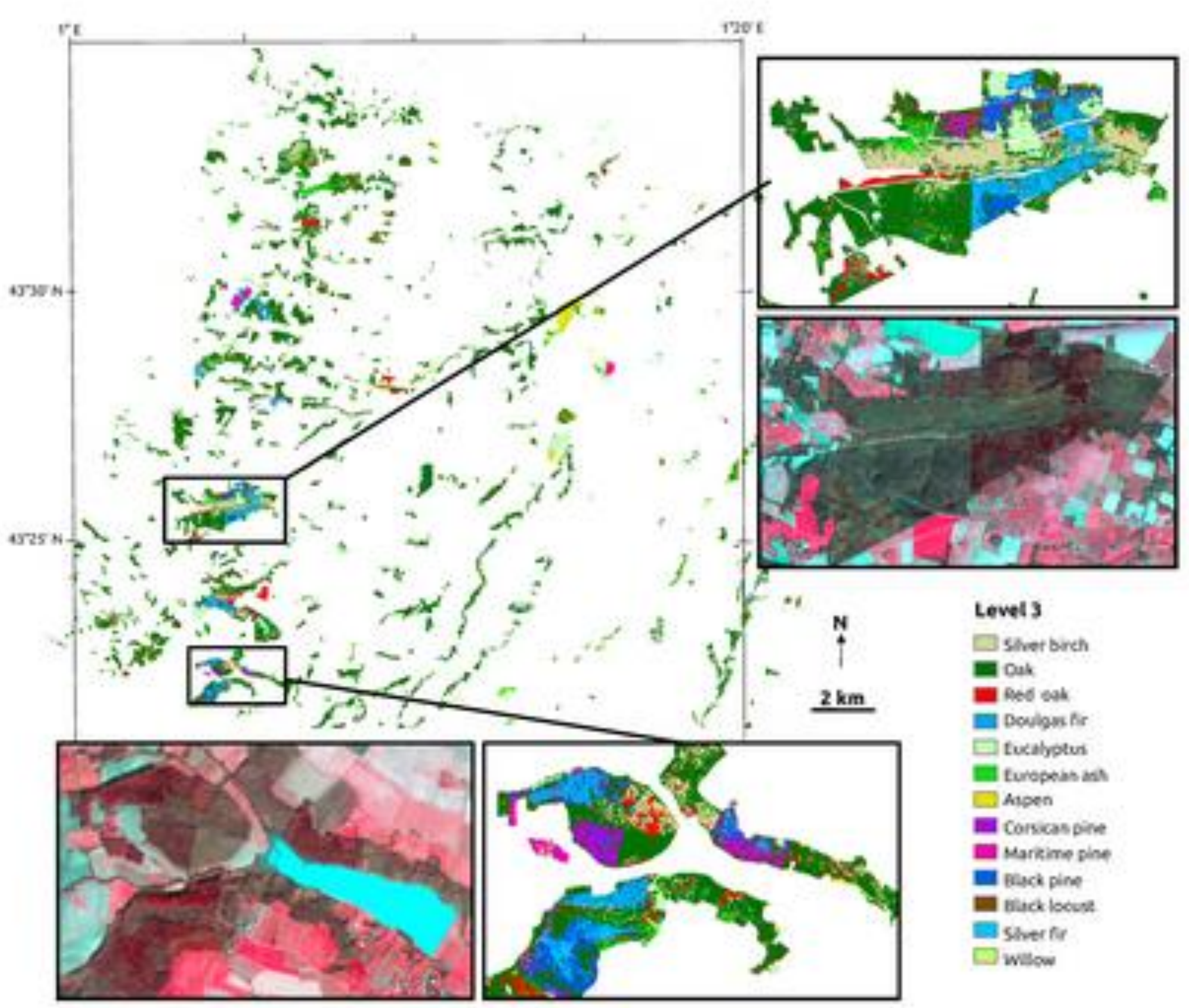


Figure 1. Classification of the Satellite Data

Classification

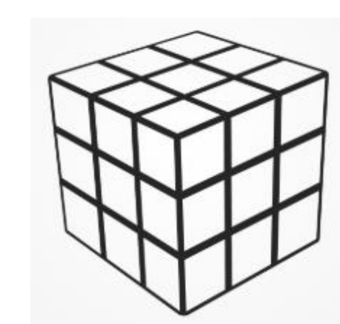
To classify the spectroscopic data, we use the **Bayes Classification Rule**, given by:

$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{(n/2)}|\Sigma|(1/2)} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

where x is the pixel to be classified and **n** is the number of dimensions of x

The Model

We model the data as $\mathbf{vec}(\mathbf{Y}) \sim N(\mu, \Sigma)$,
where $\mathbf{vec}(\mathbf{Y})$ represents the pixel (taking into account the spectra and time sampling) modeled as **Normal distribubtion** with mean μ, and covariance matrix Σ.
The covariance matrix Σ can be estimated as:



Full model (Naïve)

Number of parameters is:
$pd + \frac{pd\,(pd+1)}{2}$, which is huge,
where p is the no. of spectra and d is the number of time intervals.

Parsimonious model

. $\Sigma = \Sigma_S \otimes \Sigma_T$

Kronecker product of Σ_S and Σ_T,
where covariance of spectra Σ_S is a dXd matrix and covariance of time Σ_T is a pXp matrix.

Number of parameters reduces to:
$pd + \frac{d\,(d+1)+p\,(p+1)}{2}$.

We can assume:

Independent model

- Spectrum independence
- Time independence

$$\Sigma_S, \Sigma_T = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & [\cdots] & 0 \\ 0 & 0 & \sigma_{d,p} \end{bmatrix}$$

Kernel model

- Gaussian
- Exponential
- Quadratic
- Circular
- Uniform

Unknown model

When distribution of both Σ_S and Σ_T is unknown, we use **Flip-Flop Algorithm,** using Maximum Likelihood Estimation until convergence of Σ_S and Σ_T.

The Problem

- **Irregular temporal sampling of the pixels** – pixels of entire region do not have same moments of acquisition.
- **Missing data** – due to the presence of clouds and atmospheric disturbances.
- **High dimensional data** – 13 electromagnetic spectral bands.
- **Huge volume of data** - 6,500 billion pixels for the area of France.

We require to process and represent the data appropriately. Due to the irregular time sampling, we transform the data to regular sampling by linear interpolation – which allows us to use is as a matrix valued model.

Hence, develop models for the simulation and classification of data.

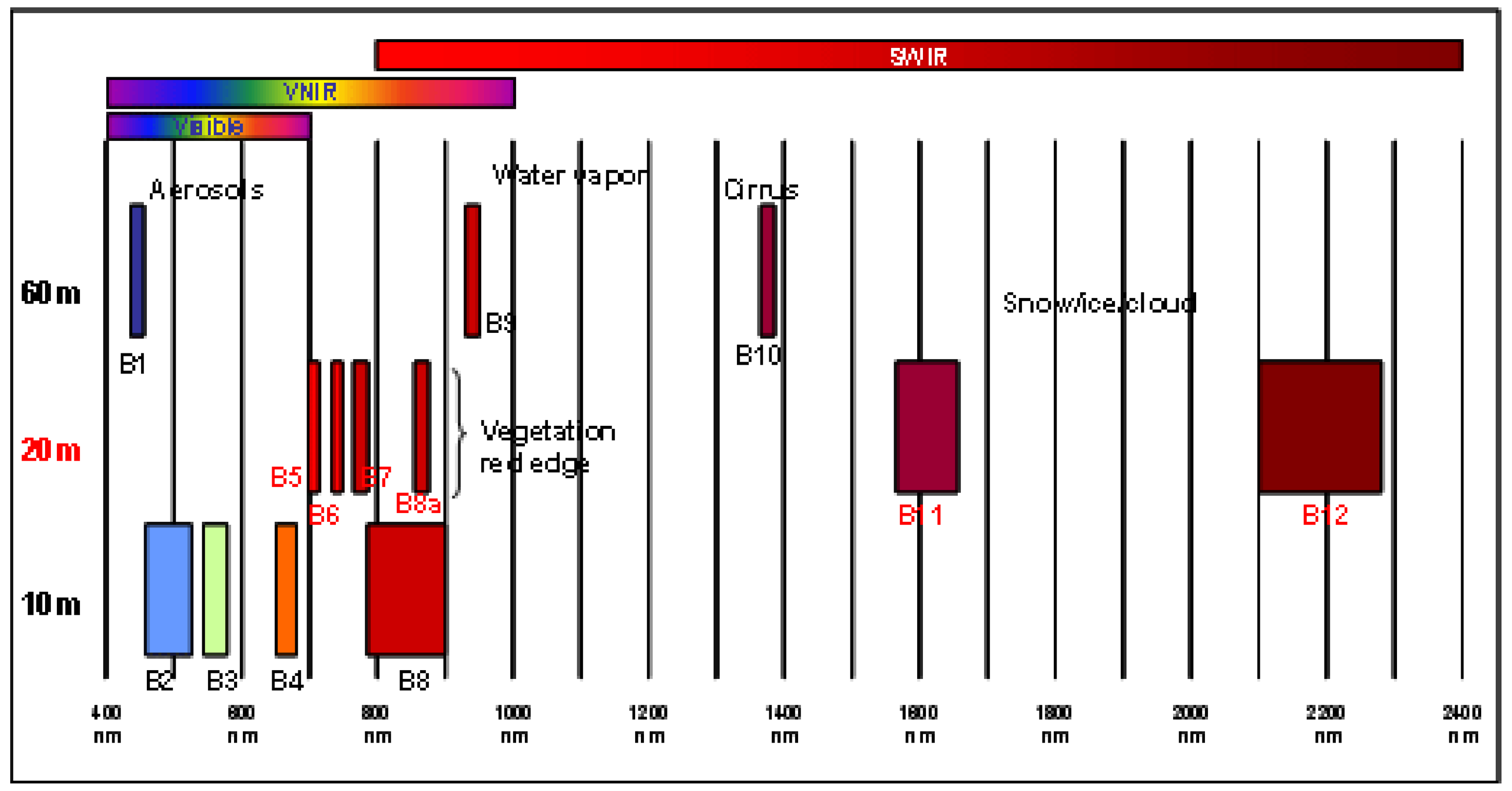


Figure 2: Sentinel-2 spectral bands

R Package

- Developed using **S4** objects and methods.
- Dependencies:
  - stats
  - graphics
  - h5
  - methods
  - mvtnorm
  - matrixcalc
  - roxygen2

The code is available at: ***https://github.com/asmitapoddar/BayesSentinel***

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References

1. Charles Bouveyron and Camille Brunet-Saumard. Model-based clustering of high-dimensional data: A review. Computational Statistics & Data Analysis, 71:52–78, 2014.
2. Nelson Lu and Dale L Zimmerman. The likelihood ratio test for a separable covariance matrix. Statistics & probability letters, 73(4):449–457, 2005.
3. Muni S Srivastava, Tatjana von Rosen, and Dietrich Von Rosen. Models with a kronecker product covariance structure: estimation and testing. Mathematical Methods of Statistics, 17(4):357–370, 2008.
4. J. Spinnato, M. Roubaud, B. Burle, and B. Torresani. Finding EEG space-time-scale localized features using matrix-based penalized discriminant analysis. Pages 6004–6008, 2014. doi: 10.1109/ICASSP.2014.6854756. URL https://doi.org/10.1109/ICASSP.2014.6854756.